Learning nonlocal closures for inverse bremsstrahlung absorption with universal differential equations

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In ICF, inverse bremsstrahlung (IB) governs the transfer of laser energy to hot plasma. IB's preferential heating of slow electrons drives the electron energy distribution function towards a super-Gaussian through the Langdon effect. Matte's formula for the super-Gaussian coefficient m over-inhibits heat transport due to lack of memory. We present a hidden state universal differential equation (UDE) for temperature evolution trained on 0D VFP simulations. The model is dimensionless, ensuring generalisation to various scales, and constrained to recover behaviour in the Maxwellian limit. A further neural network predicts thermal conductivity from the hidden state more accurately than the Matte model and enables a simple coupling with the SNB nonlocal model.

ADEPT: Auto-Differentiation Enabled Plasma Transport [2]

ADEPT (`github.com/ergodicio/adept`) offers end-to-end differentiable simulation of plasma transport PDE's in JAX. It supports state-of-the-art recording and cataloging of simulations with mlflow. Current equation sets include EPW, two-fluid, Vlasov (1D2V and 2D2V) and Vlasov-Fokker-Planck (VFP) in 1D1V using spherical harmonics. Solver methods include operator splitting and exponential integration powered by `diffrax`.

Semi-implicit bilinear electron–electron collisions

The isotropic Fokker–Planck–Landau operator is solved semi-implicitly in the energy variable $\varepsilon = \frac{1}{2}m_e v^2$,employing the symmetric weak (bilinear) form of Buet & Le Thanh [1]

$$C_{\mathrm{ee0}}(f)^{(n+1)} \sim \frac{1}{\sqrt{\varepsilon}}\frac{\partial}{\partial\varepsilon}\left[\int K\frac{\partial f'^{(n)}}{\partial\varepsilon'}\,\mathrm{d}\varepsilon'\, f^{(n+1)} + \int K\, f'^{(n)}\,\mathrm{d}\varepsilon'\,\frac{\partial f^{(n+1)}}{\partial\varepsilon}\right] = \frac{1}{\sqrt{\varepsilon}}\frac{\partial}{\partial\varepsilon}\left[C(f)f + D(f)\frac{\partial f}{\partial\varepsilon}\right]$$

where the kernel $K(\varepsilon,\varepsilon') = \min(\varepsilon^{3/2},\varepsilon'^{3/2})$. Chang-Cooper differencing is used to construct the tridiagonal system, which along with the bilinear symmetry of Buet's weak form ensures:

Density conservation Positivity Maxwellian preservation

Non-decreasing entropy Energy conservation as $\mathrm{d}t \to 0$

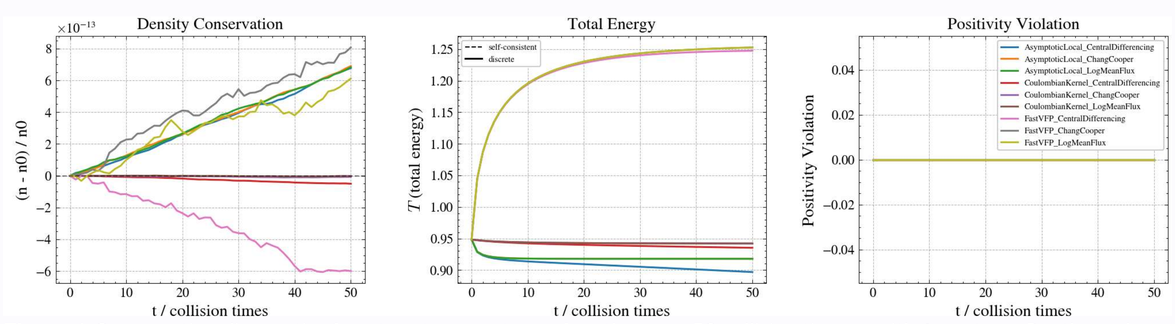

Figure 1 *Conservation diagnostics for a two-temperature relaxation over 50 collision times τ with $\mathrm{d}t = 0.1\tau$. Buet's weak form w/ CoulombianKernel (violet/brown) conserves density to machine precision, energy to 0.5%, and positivity never violated.*

Inverse bremsstrahlung: the Langdon operator

Inverse bremsstrahlung acts as an additional diffusion in velocity space $\bar{D} = D + D_{\mathrm{IB}}$:

$$D_{\mathrm{IB}} \propto \nu_{ei}^{(\mathrm{th})}\, U_p(t)\, g(\varepsilon), \qquad g(\varepsilon) = \frac{1}{1+(\nu_{ei}(\varepsilon)/\omega_0)^2} \implies \frac{\partial T_{\mathrm{e}}}{\partial t} = \frac{8}{9\sqrt{\pi}}\nu_{ei}^{(\mathrm{th})} U_p \frac{\int_0^\infty g'\, f\,\mathrm{d}\varepsilon}{f^{(\mathrm{mb})}(v=0)},$$

where $U_p = \frac{1}{2}m_e v_{\mathrm{osc}}^2$ is the ponderomotive (quiver) energy and $g' = \partial g/\partial\varepsilon$ is a weighting function peaked near $\nu_{\mathrm{ei}}(\epsilon^*) = \omega_0$. In the absence of collisions, f will approach a super-gaussian distribution $\sim\exp[-(v/v_T)^m]$, $m \to 5$ gradually reducing absorption as the bulk flattens.

The quasi-static Langdon & Matte models

Langdon [3] and Matte et al. [4] respectively provide simple models for the absorption rate and super-Gaussian parameter m under inverse bremsstrahlung in terms of the Langdon parameter $\alpha = ZU_p/k_B T_e$ in the high-frequency limit $\nu^* \to 0$:

$$\frac{\int_0^\infty g'\, f\,\mathrm{d}\varepsilon}{f^{(\mathrm{mb})}(v=0)} \to \frac{f(v=0)}{f^{(\mathrm{mb})}(v=0)} = 1 - \frac{0.553}{1+0.27/\alpha}, \quad m(\alpha) = 2 + \frac{3}{1+1.66/\alpha^{0.724}}.$$

These models have no memory due to their assumption that the distribution function instantaneously conforms to the shape predicted by the Langdon parameter. This assumption is most severe at the beginning of the pulse and between picket and main pulse. In reality the shape of the distribution will lag the drive. Furthermore, they do not properly capture finite frequency effects.

A memory-carrying Universal Differential Equation (UDE)

We introduce a dimensionless hidden state model M which represents the ratio $\int_0^\infty g'\, f\,\mathrm{d}\varepsilon / \int_0^\infty g'\, f^{(\mathrm{mb})}\,\mathrm{d}\varepsilon$ (i.e. $M = 1 \implies$ Maxwellian) and learn a closure for its evolution:

$$\frac{\partial T_{\mathrm{e}}}{\partial t} = \frac{8}{9\sqrt{\pi}}\nu_{ei}^{(\mathrm{th})} U_p G(\nu^*)\, M, \quad \frac{\partial M}{\partial t} = \nu_{ei}^{(\mathrm{th})}\Big[-(M-1)\, D_\theta(M,\nu^*) + \frac{2U_p}{T}G(\nu^*)\, A_\theta(M,\nu^*)\Big],$$

where $G = \int_0^\infty g'\, f\,\mathrm{d}\varepsilon$ is calculated on the fly with quadrature

$D_\theta = \mathrm{softplus}(\text{2-16-1 MLP}) \geq 0$: free relaxation rate, vanishing effect at $M = 1$.

$A_\theta = \exp[-(1-M)\,\mathrm{softplus}(\text{2-16-1 MLP})]$, ensuring exact Maxwellian heating rate at $M = 1$.

Training: least-squares through the solver

Data: a dimensionless VFP sweep, 10 collisionalities × 17 pulse families × 120 snapshots; a picket held out.

Objective: $\sum(T_{\mathrm{model}-T_{\mathrm{sim}}})^2$

Solver: Levenberg–Marquardt + forward-mode AD through the implicit-Euler `diffrax` solver.

Results vs the Matte model

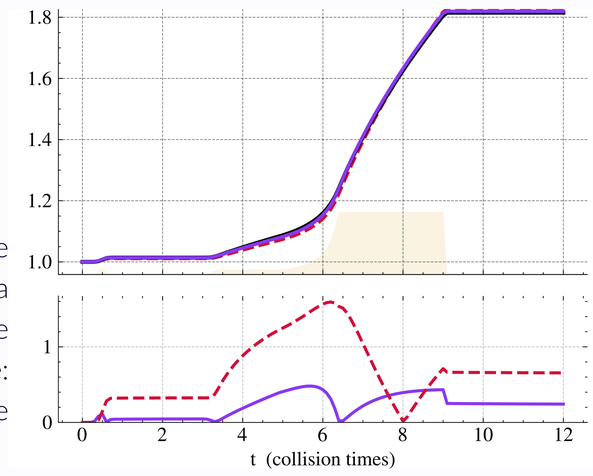

Figure 2 *Held-out single-picket isentropic-rise pulse, mid collisionality ($\nu^{*\mathrm{th}}$) ≈ 0.024). The Matte model's quasi-static error peaks at the isentropic rise (∼1.6%); the free one-state closure stays ≲0.7%.*

—— *VFP truth*
—— *closure*
····· *Matte.*

Trained on $T(t)$ alone across the full sweep, the latent M rediscovers the true g'-moment with a correlation of 99%. The free one-state closure beats the memoryless Matte model everywhere: median T-error 0.19% vs 1.03%, 99th percentile 1.07% vs 2.52%.

Predicting thermal conductivity

Inhibition of thermal conductivity $\kappa^{(\mathrm{IB})}/\kappa^{(\mathrm{SH})}$ can be computed by integrating f and regressed against M and ν^* to train a small neural network $R(M,\nu^*)$ (2-16-1 MLP).

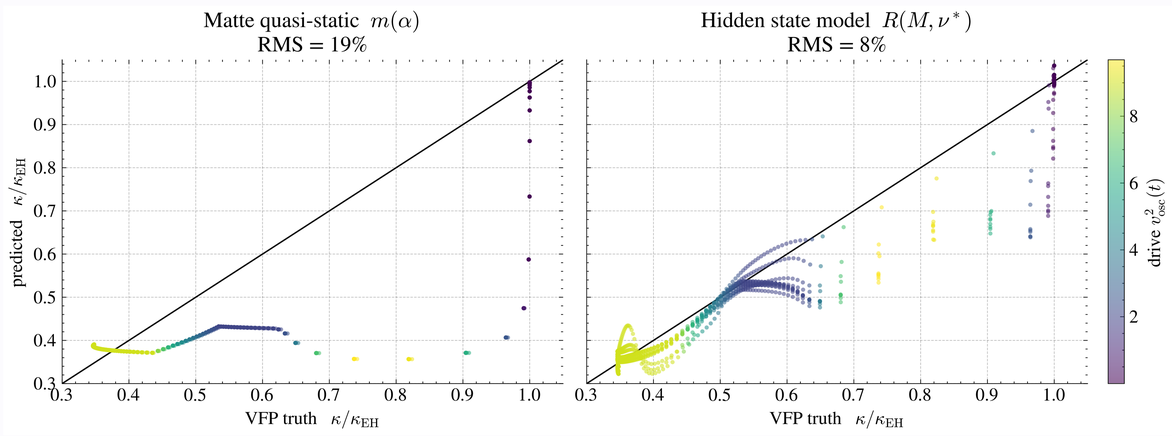

Figure 3 *Conductivity reduction on the held-out picket at various initial collisionalities ν^*: the quasi-static order (left) gives RMS 19%; the learned mapping $R(M,\nu^*)$ (right) tracks the VFP truth to 8%.*

Coupling with SNB: nonlocal super-gaussian transport

As M represents a low velocity moment of f, our model may still perform adequately in multi-dimensions without re-training. We test this hypothesis by simulating the heating of a $n_e = 1.5 \times 10^{-19}\mathrm{cm}^3$ nitrogen plasma by a Gaussian laser beam with a peak intensity of $6.3 \times 10^{13}\mathrm{W/cm}^2$ and FWHM of 150μm in 1D planar geometry. We find that approximating VFP requires accounting for the Langdon/Matte effect and nonlocal heat transport, which we capture by including a modified IB source term in the SNB model.

$$\left[\frac{1}{\lambda_{\mathrm{ee}}} - \nabla\cdot\frac{\lambda\mathrm{ei}}{3}\right]H_g = -\nabla\cdot w_g\frac{\kappa^{(\mathrm{IB})}}{\kappa^{(\mathrm{SH})}}Q^{(\mathrm{SH})}$$

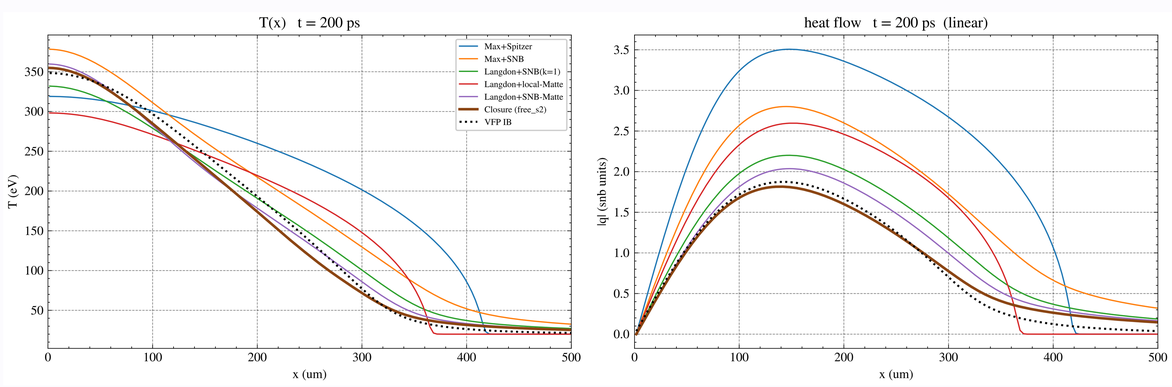

Figure 4 *1D temperature ramp at 200 ps: $T(x)$ (left) and heat flow (right). Our closure (thick brown) tracks the VFP-IB truth (dotted) in both when incorporated into the SNB model. Neglecting the Langdon effect on absorption ("Max+...") overpredicts the rate of heating.*

References

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Disclaimer

This material is partially based upon work supported by the DOE Office of Fusion Energy under Award Number(s) DE-SC0024863. This research used resources of the National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the US Department of Energy under Contract No. DE-AC02-05CH11231 using NERSC award FES-ERCAP0026741.